Errors to Avoid

As we have said, the integral of f from a to b will give us the **area under the curve (between the x-axis and the curve) when f (x) is non-negative.** $A = \int_a^b f(x)\,dx$

When f is negative, the value of the integral will be negative.

Read this next part carefully so that you will be able to avoid this very common error!!!

Area is always positive but the value of an integral does not have to be positive – it may be negative if the majority of the region that you are integrating through lies below the x-axis.

That means that when the curve is below the x-axis $A = -\int_a^b f(x)\,dx$. In other words, the area will be the opposite of the value of the integral. Or it might make more sense to say that the value of the integral will be the negative of the area of the region:

$\int_a^b f(x)\,dx = -A$ when the region with area A lies below the x-axis

That is an important distinction to make! When you are asked to find the **Total area**, or Total Distance traveled, you will have to make sure you do not have areas under the x-axis that would be negative and cancel out the positive area. You will have to add the absolute values of the different area sections.

When $f(x) \geq 0$, the value of the integral is positive and equal to the area.
When $f(x) < 0$, the value of the integral is negative and equal to the negative of the area.

Using that knowledge about how the position of the curve above or below the x-axis affects the signs of the integral we can actually determine the value of some integrals by just looking at the graph.

For instance, $\int_0^{\pi} \sin x\,dx$ cannot be found in your head. If you graph y = sin x between 0 and π you will see that the curve is a single "hump" above the x-axis and the area of that region will require methods that we have not learned yet. But, we can easily find the value of the integral $\int_{-\pi}^{\pi} \sin x\,dx$. It can be done in our head if we recall what we just said about how a region that lies below the x-axis will have an integral with a negative value. Graph $\int_{-\pi}^{\pi} \sin x\,dx$ and you will see that you have matching regions above (from 0 to π) and below (from -π to 0) the x-axis. Since the region above the axis will have a positive integral value and the region below the axis has a negative integral value, and we know the two regions are equal in size, the value of the integral will be zero. The two regions are canceling each other out.

However, if you are told that you need to find **the total area between the curve and the x-axis (or the area of the region)** then you have to make sure that those positive and negative regions do not cancel out. Split the integral into two parts and either take the absolute value of each part and add them together, or, as in this example, find the area of one region and double it. Either way, you need to make sure that you indicate from the beginning of the problem what method you are using (you can't magically "fix" the area in the last step!).

In summary:

The value of the integral might be positive or negative, depending on whether or not the majority of the graphed region lies above or below the x-axis.

The area of a region is always positive.

You must read the instructions of a problem carefully to see if you are finding area of a region (which by definition is positive), or using area to find the value of the integral (which might be positive or negative).